NLTS Hamiltonians from good quantum codes

Chinmay Nirkhe (IBM Research)*
joint with Anurag Anshu (Harvard)
& Niko Breuckmann (Bristol)

* prev. Berkeley
SNAKE!  WALL!  SPEAR!  TREE!

...
**SNAKE!**  **WALL!**  **SPEAR!**  **TREE!**

... 

**ELEPHANT!**
SNAKE! WALL! SPEAR! TREE!

ELEPHANT!
SNAKE! WALL! SPEAR! TREE!

...↓

ELEPHANT!
Snake! Wall! Spear! Tree!

... 

Elephant!

Snake! Wall! Spear! Tree!

... 

Elephant!
QUANTUM ZOO
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NLTS Hamiltonians from good quantum codes

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Understanding classical proofs
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$NP = \text{the class of all efficiently (poly}(n)\text{ time) checkable proofs.}$

$NP$ has complete problems such as Constraint Satisfaction Problems (CSPs).
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0 1 1 0 1 ... 0 1
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$Ci : \{0,1\}^3 \rightarrow \{0,1\}$

Local check $C_i = x_1 \oplus x_2 \oplus x_3 = 0$.  

$C_i$ not necessarily geometrically local
Understanding classical proofs

NP = the class of all efficiently (poly(n) time) checkable proofs.

NP has complete problems such as Constraint Satisfaction Problems (CSPs).

\[
\begin{array}{cccc}
0 & 1 & 1 & 0 & 1 \\
0 & 1 & \ldots & 0 & 1
\end{array}
\]

local check \( C_i = x_1 \oplus x_2 \oplus x_3 = 0 \).

\[
C_i : \{0, 1\}^3 \rightarrow \{0, 1\}
\]

\[
C : \{0, 1\}^n \rightarrow [0, m] \quad \text{by} \quad C(x) = \sum_{i=1}^{m} C_i(x)
\]
**Understanding classical proofs**

NP = the class of all efficiently \(\text{poly}(n)\) time checkable proofs.

NP has complete problems such as Constraint Satisfaction Problems (CSPs).

\[
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 0 & 1 \\
\end{array}
\quad \ldots \quad
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0 & 0 & 1 \\
\end{array}
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Local check \(C_i = x_1 \oplus x_2 \oplus x_3 = 0\).

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C_i : \{0,1\}^3 \rightarrow \{0,1\}
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C : \{0,1\}^n \rightarrow [0,m] \quad \text{by} \quad C(x) = \sum_{i=1}^{m} C_i(x)
\]

Decide if

1. \(\exists x, C(x) = 0\).
2. \(\forall x, C(x) \geq 1\).
Two extensions of the notion of proofs

NP ↓ GMA
Two extensions of the notion of proofs

\[ \text{NP} \xrightarrow{GMA} \text{QMA} \]

q. ph. so they require a q. verifier (BQP)
Two extensions of the notion of proofs

NP → GMA

QMA

Calculating ground energy of local Hamiltonians is a complete problem
Two extensions of the notion of proofs

\[ \text{NP} \quad \text{GMA} \]

Calculating ground energy of local Hamiltonians is a complete problem

\[ h_i = \text{linear local operator calculating energy} \]

\[ h_i = 1000\langle 000| + 111\rangle\langle 111| \]
Two extensions of the notion of proofs

- **QMA**
- **GMA**

Calculating ground energy of local Hamiltonians is a complete problem

\[ h_i = \text{linear local operator calculating energy} \]

\[ H = \sum_{i=1}^{m} h_i \quad |\psi\rangle \rightarrow \langle \psi | H | \psi \rangle \text{ (energy)} \]
Two extensions of the notion of proofs:

- **QMA**
  
  - $h_i = \text{linear local operator calculating energy}$
  
  - $h_i = 1000\langle 000| + 111\rangle\langle 111|$ 
  
  - $H = \sum_{i=1}^{m} h_i$

- **NP**
Two extensions of the notion of proofs

\[ h_i = \text{linear local operator calculating energy} \]

\[ H = \sum_{i=1}^{m} h_i \]

\[ \langle \Psi | H | \Psi \rangle \rightarrow \text{(energy)} \]

\[ \lambda_{\text{min}}(H) = \min_{|\Psi\rangle} \langle \Psi | H | \Psi \rangle \]

\[ h_i = 100 \langle 000 | + |111 \rangle \langle 111 | \]

NP

GMA
Two extensions of the notion of proofs

\[ h_i = \text{linear local operator calculating energy} \]

\[ H = \sum_{i=1}^{m} h_i \]

\[ |\psi\rangle \rightarrow \langle \psi | H | \psi \rangle \quad \text{(energy)} \]

ground energy \[ \lambda_{\min}(H) = \min_{|\psi\rangle} \langle \psi | H | \psi \rangle \]

QMA-hard to decide for \[ b-a = 1/\text{poly}(m), \]

1. \[ \lambda_{\min}(H) \leq a \iff \exists |\psi\rangle, \langle \psi | H | \psi \rangle \leq a \]
2. \[ \lambda_{\min}(H) \geq b \iff \forall |\psi\rangle, \langle \psi | H | \psi \rangle \geq b \]
Two extensions of the notion of proofs

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$\Rightarrow$ ground states of local Hamiltonians are a “canonical” form for all q. ps.
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Two extensions of the notion of proofs

QMA-hard to decide for $b - a = 1 / \text{poly}(m)$,
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It's widely believed that $\text{NP} \neq \text{QMA}$

Therefore, not all ground states of local Hamiltonians can be classically described (in an efficiently verifiable manner)
Two extensions of the notion of proofs

- NP
  - GMA
  - PCPs
Two extensions of the notion of proofs

we think of pfs as requiring step-by-step checking.
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PCP theorem: Every NP problem (i.e., every pf) can be converted into a form s.t. only $O(1)$ bits need to be read to be 99% confident in validity.
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NP-hard to decide if

$1 \exists x, C(x) = 0$

$2 \forall x, C(x) \geq \frac{m}{2}$ (prev. 1)
Two extensions of the notion of proofs

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NP-hard to decide if $C(x)$: analog of $\langle \psi | H | \psi \rangle$

1. $\exists x, C(x) = 0$
2. $\forall x, C(x) \geq \frac{m}{2}$ (prev. 1)

Important consequence: Noisy pfs suffice!
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[C(x) = analog of $\langle \psi | H | \psi \rangle$]

Important consequence: Noisy pfs suffice!

Any $x$ s.t. $C(x) < \frac{m}{4}$ can
be prob. verified with $O(1)$ queries.
The Quantum Prob. Checkable Pts. Conjecture

NP \{ QMA \}
   \{ PCP_s \}
   \{ QPCP_s \}
The Quantum Prob. Checkable Pts. Conjecture

Conjecture: Every QMA problem (i.e. quantum pf.) can be converted into a form s.t. only $O(1)$ qubits need to be measured.
The Quantum Prob. Checkable Pts. Conjecture

Conj. For $\epsilon > 0$, it's QMA-hard to decide:

1. $\exists |\psi\rangle$ s.t. $\langle\psi|H|\psi\rangle = 0$ (moranally)
2. $\forall |\psi\rangle$, $\langle\psi|H|\psi\rangle \geq \epsilon m$
The Quantum Prob. Checkable Pfs. Conjecture

Conj. For $\varepsilon > 0$, it’s QMA-hard to decide:
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Conjecture: Every QMA problem (i.e. quantum pf.) can be converted into a form s.t. only $O(1)$ qubits need to be measured.

Similar to PCP theorem, every state of energy $\leq \frac{\varepsilon}{2}m$ is a valid pf. for a QPCP local Hamiltonians.

Set of pfs is much larger!
An important consequence of QPCPs

A) (if NP ≠ QMA) quantum pfs. cannot be classically described (in any efficiently checkable manner)

B) low energy states of Q2PCP local Hamiltonians are also valid pfs (since they are noisy pfs.)
An important consequence of QPCPs

(A) (if NP $\neq$ QMA) quantum pfs. cannot be classically described (in any efficiently checkable manner)

(B) low energy states of Q2PCP local Hamiltonians are also valid pfs (since they are noisy pfs.)

$\Rightarrow$ There exist local Hamiltonians such that every low energy state cannot be classically described
An important consequence of QPCPs

A) (if NP ≠ QMA) quantum pfs. cannot be classically described (in any efficiently checkable manner)

B) low energy states of Q2PCP pfs (since they are noisy pfs.)

⇒ There exist local Hamiltonians such that every low energy state cannot be classically described

Constant depth q. circuit descriptions are classically checkable pfs. for output state
An important consequence of QPCPs

A. (if $\text{NP} \neq \text{QMA}$) quantum pfs. cannot be classically described (in any efficiently checkable manner)

B. low energy states of QPCP pfs. (since they are noisy pfs.) local Hamiltonians are also valid pfs.

$\Rightarrow$ There exist local Hamiltonians such that every low energy state cannot be classically described

Constant depth q. circuit descriptions are classically checkable pfs. for output state.

No low energy trivial states. There exist local Hams. s.t. no low-energy state is the output of a constant depth circuit.

[Freedman- Hastings ’14]
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- If it was false, then QPCP would have been trivially false.
- Makes a statement about physically realizable robust entanglement.
No low energy trivial states There exist local Hams. s.t. no low-energy state is the output of a constant depth circuit. [Freedman-Hastings 14]

- If it was false, then QPCP would have been trivially false.
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Theorem [Anurag Anshu, Niko Breuckmann, & C.N. ‘22]

Local Hamiltonians corresponding to most* linear-rate and -distance QLDPC error-correcting codes are NLTS Hamiltonians.
No low energy trivial states. There exist local Hamiltonians such that no low-energy state is the output of a constant depth circuit.

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- If it was false, then QPCP would have been trivially false.
- Makes a statement about physically realizable robust entanglement.

Theorem [Anurag Anshu, Niko Breuckmann, & C.N. ‘22]

Local Hamiltonians corresponding to most linear-rate and -distance QLDPC error-correcting codes are NLTS Hamiltonians.

\[ \exists \varepsilon > 0, \text{ and Hamiltonian family } \mathcal{H} \text{ s.t. every state } \psi \text{ of energy } \leq \varepsilon N, \text{ the minimum depth circuit to generate } \psi \text{ is } \Omega(\log n). \]
Proof sketch of the NLTS theorem

1. Trivial states $\Rightarrow$ Local Hamiltonians
   $\Rightarrow$ Circuit depth lower bounds

   Lightcone for
   low depth circuits
Proof sketch of the NLTS theorem

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   Light cones for low depth circuits

2. Error Correction Codes (ECC)

   Low energy subspace of expanding codes.
Proof sketch of the NLTS theorem

1. Trivial states $\Rightarrow$ Local Hamiltonians
   $\Rightarrow$ Circuit depth lower bounds

2. Error Correction Codes (ECC)
   - Low energy subspace of expanding codes

3. Erasure errors for quantum codes
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a g. circuit of depth $t$, then $U^*AU$ is a $\leq 2^t \cdot |A|$ local operator.
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a g. circuit of depth $t$, then $U^\dagger AU$ is a $\leq 2^t \cdot |A|$ local operator.
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a $g$ circuit of depth $t$, then $U^*AU$ is a $\leq 2^t |A|$ local operator.
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a $g$ circuit of depth $t$, then $U^t A U$ is a $\leq 2^t |A|$ local operator.
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a $g$ circuit of depth $t$, then $U^t A U$ is a $\leq 2^t |A|$ local operator.

Given a local Hamiltonian $H = \sum_i h_i$ and a state $|\psi\rangle = U |0^n\rangle$, we can evaluate $\langle \psi | H | \psi \rangle$ in classical time $2^{2^t \cdot \text{poly}(n)} = \text{poly}(n)$ when $t = O(1)$. 

$\leq 2^t |A|$
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a $g$ circuit of depth $t$, then $U^\dagger AU$ is a $\leq 2^t |A|$ local operator.

Given a local Hamiltonian $H = \sum_i^m h_i$ and a state $|\psi\rangle = U|0^n\rangle$, we can evaluate $\langle \psi | H | \psi \rangle$ in classical time $2^{2t} \cdot \text{poly}(n) = \text{poly}(n)$ when $t = O(1)$.

\[
\langle \psi | H | \psi \rangle = \sum_i^m \langle \psi | h_i | \psi \rangle = \sum_i^m \langle 0^n | U^\dagger h_i U | 0^n \rangle
\]
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a $q$ circuit of depth $t$, then $U^t A U$ is a $\leq 2^t |A|$ local operator.

Given a local Hamiltonian $H = \sum_i h_i$ and a state $|\psi\rangle = U|0^n\rangle$, we can evaluate $\langle \psi | H | \psi \rangle$ in classical time $2^{2t} \cdot \text{poly}(n) = \text{poly}(n)$ when $t = O(1)$.

$$\langle \psi | H | \psi \rangle = \sum_i \langle \psi | h_i | \psi \rangle = \sum_i \langle 0^n | U^\dagger h_i U | 0^n \rangle$$

computation on $O(2^t)$ qubits
Lightcones and quantum circuits

If $A$ is a local operator and $U$ is a $t$-gate circuit of depth $t$, then $U^t AU$ is a $\leq 2^t \cdot |A|$ local operator.

Given a local Hamiltonian $H = \sum_i h_i$ and a state $|\psi\rangle = U|0^n\rangle$, we can evaluate $\langle \psi | H | \psi \rangle$ in classical time $2^t \cdot \text{poly}(n) = \text{poly}(n)$ when $t = \text{O}(1)$.

$$
\langle \psi | H | \psi \rangle = \sum_i \langle \psi | h_i | \psi \rangle
= \sum_i \langle 0^n | U^t h_i U | 0^n \rangle
$$

Low-depth states are classical witnesses for energy computation on $O(2^t)$ qubits
Trivial states $\Rightarrow$ Local Hamiltonians

The state $|0\rangle$ is the unique solution to a very simple local Hamiltonian.
Trivial states $\implies$ Local Hamiltonians

The state $|0^m\rangle$ is the unique solution to a very simple local Hamiltonian.

$$H_0 = \sum_{i=1}^{\hat{n}'} |1\rangle\langle 1|_i \leftarrow \text{qubit-wise projectors enforcing qubits equal } 10\rangle.$$
Trivial states $\Rightarrow$ Local Hamiltonians

The state $|0^{n'}\rangle$ is the unique solution to a very simple local Hamiltonian.

$$H_0 = \sum_{i=1}^{n'} |1\rangle\langle 1|_i \Rightarrow \text{qubit-wise projectors enforcing qubits equal } 10\rangle.$$  

$H_0$ is commuting and has a spectrum of $0, 1, 2, \ldots, n'$, with eigenvectors $|x\rangle$ of eigenvalue $|x1\rangle$. 
Trivial states $\Rightarrow$ Local Hamiltonians

The state $|0^{m'}\rangle$ is the unique solution to a very simple local Hamiltonian.

$$H_0 = \sum_{i=1}^{m'} |1\rangle\langle 1|_i \quad \leftarrow \text{qubit-wise projectors enforcing qubits equal } 10^\rangle.$$  

$H_0$ is commuting and has a spectrum of $0, 1, 2, \ldots, n'$, with eigenvectors $|x\rangle$ of eigenvalue $1 \times 1$.

Let $H_u = U^\dagger H U$ for depth $t$ circuit $U$. 

**Trivial states \(\Rightarrow\) Local Hamiltonians**

The state \(|0^{n'}\rangle\) is the unique solution to a very simple local Hamiltonian.

\[
H_0 = \sum_{i=1}^{n'} |1\rangle\langle 1|_i \quad \leftarrow \text{qubit-wire projectors enforcing qubits equal } 10\rangle.
\]

\(H_0\) is commuting and has a spectrum of \(0, 1, 2, \ldots, n'\), with eigenvectors \(1x\rangle\) of eigenvalue \(1x\).

Let \(H_u = U^t H U\) for depth \(t\) circuit \(U\).

\(H_u\) is commuting and has a spectrum of \(0, 1, 2, \ldots, n'\), with eigenvectors \(U1x\rangle\) of eigenvalue \(1x\).

And \(H_u\) is a \(2^t\)-local Hamiltonian.
Local indistinguishability

Two states $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$, \[ \Psi_S = \Psi'_S. \]
**Local indistinguishability**

Two states $|\psi\rangle$ and $|\psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$,

$$\psi_{-S} = \psi'_{-S}$$

Ex. The states $|\psi\rangle = \frac{10^n\rangle \pm |1^n\rangle}{\sqrt{2}}$ are $(n-1)$ locally indistinguishable.
Local indistinguishability

Two states $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$, $\Psi_S = \Psi'_S$.

Ex. The states $|\psi_{\pm}\rangle = \frac{|0^n\rangle \pm |1^n\rangle}{\sqrt{2}}$ are $(n-1)$ locally indistinguishable.

Any strict reduced density matrix equals

\[
\left(\begin{array}{c|cc|cc}
\vdots & 0 \times 0 & |1\rangle \times |1\rangle & \vdots \\
\hline
|0\rangle \times |0\rangle & |0\rangle \times |0\rangle & |1\rangle \times |1\rangle & |1\rangle \times |1\rangle \\
\hline
|1\rangle \times |0\rangle & |1\rangle \times |0\rangle & |1\rangle \times |1\rangle & |1\rangle \times |1\rangle \\
\end{array}\right)_{S} = \frac{|0\rangle \times |0\rangle |0\rangle \times |1\rangle + |1\rangle \times |0\rangle |1\rangle \times |1\rangle}{2}.
\]
Local indistinguishability

Two states $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$, $\Psi_S = \Psi'_S$. 
Local indistinguishability

Two states $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$, $\Psi_S = \Psi'_S$.

Lemma: If $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable, then if $|\Psi\rangle = U|0^n\rangle$ for $U$ of depth $t$, then $2^t \geq d$. \( \Rightarrow \) $t \geq \log d$. 
Local indistinguishability

Two states $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$,

$$\Psi_S = \Psi'_S.$$ 

**Lemma** If $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable, then if $|\Psi\rangle = U|0^n\rangle$ for $U$ of depth $t$, then $2^t \geq d$. $\Rightarrow t \geq \log d$.

**Proof.**

$$\langle\Psi'| H_U |\Psi\rangle = \sum_i \langle\Psi'| h_i |\Psi\rangle$$

$$= \sum_i \langle\Psi| h_i |\Psi\rangle$$

since $H_U$ is $2^t$-local and are $d > 2^t$ locally indistinguishable.
Local indistinguishability

Two states $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$,

$$|\psi_S \rangle = |\psi'_S \rangle.$$

Lemma (If $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable, then if $|\Psi\rangle = U|0^n\rangle$ for $U$ of depth $t$, then $2^t \geq d$. $\Rightarrow t \geq \log d$).

Proof. $\langle \psi' | H_U | \psi \rangle = \sum_i \langle \psi' | h_i | \psi \rangle = \sum_i \langle \psi | h_i | \psi \rangle = \langle \psi | H_U | \psi \rangle = 0$

since $H_U$ is $2^t$-local and are $d > 2^t$ locally indistinguishable.
Local indistinguishability

Two states $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable if for every region $S$ of size $\leq d$,
\[ |\Psi_S\rangle = |\Psi'_S\rangle. \]

Lemma (if $|\Psi\rangle$ and $|\Psi'\rangle$ are $d$-locally indistinguishable, then if $|\Psi\rangle = U|0^n\rangle$ for $U$ of depth $t$, then $2^t \geq d$. \( \Rightarrow \) $t \geq \log d$.

Proof. $\langle \Psi' | H_U | \Psi' \rangle = \sum_i \langle \Psi' | h_i | \Psi' \rangle = \sum_i \langle \Psi | h_i | \Psi \rangle = \langle \Psi | H_u | \Psi \rangle = 0$

since $H_u$ is $2^t$-local and are $d > 2^t$ locally indistinguishable.

But groundstate $|\Psi\rangle$ is unique! $\Rightarrow |\Psi\rangle = |\Psi'\rangle$, a contradiction.
Local indistinguishability

Lemma. If $|\psi\rangle$ and $|\psi'\rangle$ are $c$-locally indistinguishable, then if $|\psi\rangle = U|0^n\rangle$ for $U$ of depth $t$, then $2^t \geq c$. \[ t \geq \log c. \]
Local indistinguishability

Lemma \( |\psi \rangle \) and \( |\psi' \rangle \) are \( d \)-locally indistinguishable, then if 
\( |\psi \rangle = U|0^n \rangle \) for \( U \) of depth \( t \), then \( 2^t \geq d \). \( \Rightarrow \) \( t \geq \log d \).

Since, spectral gap of \( H_\mu \) is 1, this argument is only robust to perturbations of \( O(\frac{1}{n}) \).
Local indistinguishability

**Lemma** If $|\psi\rangle$ and $|\psi'\rangle$ are $d$-locally indistinguishable, then if $|\psi\rangle = U|0^n\rangle$ for $U$ of depth $t$, then $2^t \geq d$. $\Rightarrow t \geq \log d$.

Since, spectral gap of $H_U$ is 1, this argument is only robust to perturbations of $O(\frac{1}{n})$.

Using mathematics from Chebyshev polynomials, we can make it robust.
Local indistinguishability

Lemma 1 If $|\psi\rangle$ and $|\psi'\rangle$ are $d$-locally indistinguishable, then if $|\psi\rangle = U|0^n\rangle$ for $U$ of depth $t$, then $2^t \geq d$. ⇒ $t \geq \log d$.

Since, spectral gap of $H_U$ is 1, this argument is only robust to perturbations of $O(\frac{1}{n})$.

Using mathematics from Chebyshev polynomials, we can make l.b. robust.

Theorem Let $S_1, S_2 \subset \{0,1\}^n$ be sets and $p(.)$ a prob. dist. on $\{0,1\}^n$. If $p(S_1), p(S_2) \geq \mu$, then minimum q. ckt. depth to generate $p$ is $\Omega \left( \log \left( \frac{\text{dist}(S_1, S_2)^2 \cdot \mu}{n} \right) \right)$.
Local indistinguishability

Theorem Let $S_1, S_2 \subseteq \{0, 1\}^n$ be sets and $p(\cdot)$ a prob. dist. on $\{0, 1\}^n$. If $p(S_1), p(S_2) \geq \mu$, then minimum q. ckt. depth to generate $p$ is $\mathcal{O}(\log(\frac{\text{dist}(S_1,S_2)^2 \cdot \mu}{n}))$. 
**Local indistinguishability**

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\[ \{0,1\}^n \]
Local indistinguishability

Theorem. Let $S_1, S_2 \subset \{0,1\}^n$ be sets and $p(\cdot)$ a prob. dist. on $\{0,1\}^n$. If $p(S_1), p(S_2) \geq \mu$, then minimum q.ckt. depth to generate $p$ is

$$\Omega \left( \log \left( \frac{\text{dist}(S_1, S_2)^2 \cdot \mu}{n} \right) \right).$$

Proof sketch. Let $|\psi\rangle$ generate $p$. 

\[\text{Diagram with sets } S_1, S_2 \subset \{0,1\}^n\]
Local indistinguishability

**Theorem** Let $S_1, S_2 \subset \{0,1\}^n$ be sets and $p(.)$ a prob. dist. on $\{0,1\}^n$. If $p(S_1), p(S_2) \geq \mu$, then minimum q. ckt. depth to generate $p$ is

$$\Omega \left( \log \left( \frac{\text{dist}(S_1, S_2)^2 \cdot \mu}{n} \right) \right).$$

**Pf sketch.** Let $|\psi\rangle$ generate $p$. Then 3 region $R$ s.t.
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**Proof sketch.** Let $|\Psi\rangle$ generate $p$.

Then 3 region $R$ s.t.

$|\Psi\rangle = "$flip sign of $|\Psi\rangle$ on $R$"

and $|\Psi\rangle$ and $|\Psi\rangle'$ are approx. locally indistinguishable.
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When $\text{dist}(S_1, S_2) \geq \omega(\sqrt{n})$ and $\mu = \Omega(1)$, we call such distributions well spread. To prove NLTS, we need to show $\exists$ a local Hamiltonians whose entire low-energy subspace induces well-spread distributions.
Expanding codes & Tanner codes

A linear code \( \subseteq \mathbb{F}_2^n \) can be expressed as \( \ker H \) for \( H \in \mathbb{F}_2^{m \times n} \).
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We can draw the adjacency graph corresponding to \( H \).
A linear code $C \subseteq \{0,1\}^n$ can be expressed as $\ker H$ for $H \in \mathbb{F}_2^{m \times n}$.

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If the graph is small-set expanding, $\Gamma(A) \geq (1 - \gamma)d|A|$ for all $|A| \leq c_2 n$, then the low-energy subspace of the code clusters into far-apart regions.
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\[ (H) \begin{pmatrix} x \end{pmatrix} = (0) \]
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- $\{0,1\}^n$
- $\mathbb{F}_2$
- $\mathcal{C}$
- $\ker H$
- $H$
- $\Gamma(A)$
- $\left| A \right|$
- $\gamma$
- $c_2$

\begin{align*}
( H ) ( x ) &= ( 0 )
\end{align*}
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For all \( y \in \mathbb{F}_2^n \) s.t. \( |H y| \leq c_1 m \), then either

1. \( |y| \leq c_1 \cdot c_2 n \) or
2. \( |y| \geq c_2 n \)
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A linear code $C \subseteq \{0,1\}^n$ can be expressed as $\ker H$ for $H \in \mathbb{F}_2^{m \times n}$.

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For all $y \in \{0,1\}^n$ s.t. $|Hy| \leq 3m$, then either

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For all $y \in \{0,1\}^n$ s.t. $|Hy| \leq \varepsilon m$, then either

1. $|y| \leq c_1 \cdot 3n$ or
2. $|y| \geq c_2 n$

Pf sketch: $A = \text{supp}(y)$, $\Gamma^+(A)$ = unique neighbors of $|A|$. $|\Gamma^+(A)| \geq (1-2\delta) d |A|$. Every check in $\Gamma^+(A)$ will flag. So $|Hy| \geq (1-2\delta) d |y|$ unless $|y| \geq c_2 n$. 

$$\begin{pmatrix} H \end{pmatrix} \begin{pmatrix} x \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix}$$
Expanding codes & Tanner codes

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The low-energy space of a code is a great support for a distribution that we hope to prove is well-spread.

• $\mathbb{F}_2^n$ = states that violate $\leq$ EM checks

• = Codewords

$\mathbb{F}_2^m$
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The low-energy space of a code is a great support for a distribution that we hope to prove is well-spread.

Only question is how to construct Hamiltonian with such property?
Quantum error correcting codes

Consider a state subject to an erasure error.
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Consider a state subject to an erasure error. If we could recover the original state then unless \( \bullet \) contains no information about the original state, this violates the no-cloning theorem.
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Error-correcting codes that are LDPC naturally have a local Hamiltonian, one that applies every local check.
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Exact codewords of codes of distance \( d \) require circuits of depth \( \Omega(\log d) \) to generate.

Error-correcting codes that are LDPC naturally have a local Hamiltonian, one that applies every local check.

How do we prove circuit depth lower bounds for the low-energy subspace of these code Hamiltonians?
Optimal-parameter CSS codes

There is a class of q. codes called Calderbank-Shor-Steane codes that correct for X-type (bit-flip) and Z-type (phase-flip) errors separately.
Optimal-parameter CSS codes

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They are constructed from two classical codes $C_x, C_z$ (w. check-matrix $H_x, H_z$) s.t. $C_x^\perp \leq C_z$ (equiv. $C_z^\perp \leq C_x$).
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$$d_z = \min_{w \in C_z^\perp} |w|_{C_x^\perp}, \quad d_x = \min_{w \in C_x^\perp} |w|_{C_z^\perp}$$

where $|w|_S = \min_{w' \in S} |w + w'|$.
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where $|w|_S = \min_{w' \in S} |w + w'|$.

$$d = \min \{d_x, d_z\}.$$
Expanding CSS codes

Similar to classical example, we consider codes that have the property that if $|H_2 y| \leq \varepsilon n$ then either

1. $|y|_{C_x^+} \leq c_1 \varepsilon n$ or
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$[0,1]^n$
Expanding CSS codes

Similar to classical example, we consider codes that have the property that if $|H_2y| \leq \varepsilon m$, then either

1. $|y|_{c_x^+} \leq c_1 \varepsilon n$ or
2. $|y|_{c_x^+} \geq c_2 n$.

And, if we consider a $\frac{\varepsilon}{200}$-low-energy state of the code's local Hamiltonian, measuring in the $Z$-basis yields a dist. 99.52 supported on
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All that remains to show is that the distribution is not 99\% concentrated on any 1 cluster.
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⇒ circuit depth lower bound.

Uncertainty principle: For sets $S,T \subseteq \{0,1\}^n$, any state $\Psi$ with dists. $D_x, D_z$

$$D_x(T) \leq 2\sqrt{1 - D_z(S)} + \sqrt{\frac{|S||T|}{2^n}}$$
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All that remains to show is that the distribution is not $0.992$ concentrated on any 1 cluster. $\Rightarrow$ dist. is well-spread ($\mu = \frac{1}{400}$) $\Rightarrow$ circuit depth lower bound.

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$$D_x(T) \leq 2\sqrt{1 - D_z(S)} + \sqrt{\frac{|S| \cdot |T|}{2^n}}$$

Assume $D_z$ is $0.992$ concentrated on some $Z$-cluster $S$. Then for any $X$-cluster $T$, $D_x(T) < 0.99 \Rightarrow$ Either $D_x$ or $D_z$ is well-spread.
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Uncertainty principle: For sets $S,T \subseteq \{0,1\}^n$, any state $\Psi$ with dists. $D_x,D_z$

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Assume $D_z$ is $99\%$ concentrated on some $Z$-cluster $S$. Then for any $X$-cluster $T$, $D_x(T) < 0.99 \implies$ Either $D_x$ or $D_z$ is well-spread.
**The uncertainty principle**

\[ |S| \leq \binom{n}{o(n)} \cdot 2^k \]

\[ \text{violates check } c_x \text{ def.} \]

Uncertainty principle: For sets \( S, T \subseteq \{0,1\}^n \), any state \( \psi \) with dists. \( D_x, D_z \)

\[ D_x(T) \leq 2\sqrt{1 - D_z(S)} + \sqrt{\frac{|S| \cdot |T|}{2^n}} \]

Assume \( D_z \) is \( \leq 0.99 \% \) concentrated on some \( Z \)-cluster \( S \). Then for any \( X \)-cluster \( T \), \( D_x(T) < 0.99 \Rightarrow \) Either \( D_x \) or \( D_z \) is well-spread.
The uncertainty principle

\[ |S| \leq \binom{n}{\Theta(n)} \cdot 2^n \leq 2^n + O(\sqrt{2^n}) \]

Assume \( D_z \) is \( 99\% \) concentrated on some \( Z \)-cluster \( S \). Then for any \( X \)-cluster \( T \),

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The uncertainty principle

\[ |S| \leq \binom{n}{O(n^n)}, \quad 2^r x \leq 2^r x + O(\sqrt{\varepsilon n}) \]

\[ |T| \leq 2^r + O(\sqrt{\varepsilon n}) \]

Uncertainty principle: For sets \( S, T \subseteq \{0,1\}^n \), any state \( \Psi \) with dists \( D_x, D_z \)

\[ D_x(T) \leq 2 \sqrt{1 - D_z(S)} + \sqrt{\frac{|S| \cdot |T|}{2^n}} \]

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\[ |S| \leq \binom{n}{r} \cdot 2^r \leq 2^r + O(\sqrt{\varepsilon n}) \]

\[ |T| \leq 2^r + O(\sqrt{\varepsilon n}) \]

Uncertainty principle: For sets \( S, T \subseteq [0,1]^n \), any state \( \psi \) with dists. \( D_x, D_z \)

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\[ |S| \leq \binom{n}{0(n)} \cdot 2^{r_X} \leq 2^{r_X} + O(\sqrt{\varepsilon} n) \]

\[ |T| \leq 2^{r_Z} + O(\sqrt{\varepsilon} n) \]

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The uncertainty principle

\[ |S| \leq \binom{n}{0(\varepsilon n)} \cdot 2^r_x \leq 2^r_x + O(\sqrt{\varepsilon} n) \]

\[ |T| \leq 2^{r_2} + O(\sqrt{\varepsilon} n) \]

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Uncertainty principle: For sets \( S, T \subseteq \{0,1\}^n \), any state \( \psi \) with \( D_x, D_2 \)

Assume \( D_2 \) is \( 0.992 \) concentrated on some \( Z \)-cluster \( S \). Then for any \( X \)-cluster \( T \), \( D_x(T) < 0.99 \implies \) Either \( D_x \) or \( D_2 \) is well-spread.
The uncertainty principle:

\[ |S| \leq \binom{n}{0(n)} \cdot 2^{\frac{n}{2}} \leq 2^{\frac{n}{2} + O(\sqrt{n} \cdot n)} \]

\[ |T| \leq 2^{\frac{n}{2} + O(\sqrt{n} \cdot n)} \]

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Conclusion of the proof

CSS code of linear-rate and linear-distance which are expanding are NCTS. Any state violating EN checks cannot be the output of a constant depth CFT.
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CSS code of linear-rate and linear-distance which are expanding are NCTS. Any state violating EN checks cannot be the output of a constant depth Cft.

QPCP conjecture implications

① Much harder to disprove QPCP now!
② We need a stronger classical ansatz for classical proofs of local Hamiltonians.